Eindexamen wiskunde B1-2 vwo 2003-II

Wortels optellen

5. $B_{10} = \frac{1}{10} \cdot (\sqrt{\tfrac{1}{10}} + \sqrt{\tfrac{2}{10}} + \sqrt{\tfrac{3}{10}} + \sqrt{\tfrac{4}{10}} + \sqrt{\tfrac{5}{10}} + \sqrt{\tfrac{6}{10}} + \sqrt{\tfrac{7}{10}} + \sqrt{\tfrac{8}{10}} + \sqrt{\tfrac{9}{10}} + \sqrt{\tfrac{10}{10}}) =$

$= \frac{1}{10} \cdot (\dfrac{\sqrt{1}}{\sqrt{10}} + \dfrac{\sqrt{2}}{\sqrt{10}} + \dfrac{\sqrt{3}}{\sqrt{10}} + \dfrac{\sqrt{4}}{\sqrt{10}} + \dfrac{\sqrt{5}}{\sqrt{10}} + \dfrac{\sqrt{6}}{\sqrt{10}} + \dfrac{\sqrt{7}}{\sqrt{10}} + \dfrac{\sqrt{8}}{\sqrt{10}} + \dfrac{\sqrt{9}}{\sqrt{10}} + \dfrac{\sqrt{10}}{\sqrt{10}})$

$= \frac{1}{10} \cdot \frac{1}{\sqrt{10}} \cdot (\sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6} + \sqrt{7} + \sqrt{8} + \sqrt{9} + \sqrt{10})$

6. Het verschil tussen de bovensom en de ondersom is gelijk aan de oppervlakte van een rechthoek met hoogte 1 en breedte $\frac{1}{10}$, dus geldt:
$$B_{10} - O_{10} = \tfrac{1}{10}$$

7. $A < B_n$

$A < \frac{1}{n\sqrt{n}} \cdot (\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{n})$

$n\sqrt{n} \cdot A < (\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{n})$

$A > O_n$

$A > \frac{1}{n\sqrt{n}} \cdot (\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{n}) - \frac{1}{n}$

$n\sqrt{n} \cdot A + \frac{n\sqrt{n}}{n} > (\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{n})$

Dus $n\sqrt{n} \cdot A < (\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{n}) < n\sqrt{n} \cdot A + \sqrt{n}$

8. De oppervlakte van A is gelijk aan $\displaystyle\int_0^1 \sqrt{x}\, dx = \left[\tfrac{2}{3} \cdot x^{\frac{3}{2}} \right]_0^1 = \dfrac{2}{3}$, dus geldt:

$10000 \cdot \sqrt{10000} \cdot \tfrac{2}{3} < \sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{10000} <$

$< 10000 \cdot \sqrt{10000} \cdot \tfrac{2}{3} + \sqrt{10000} \quad\rightarrow$

$\rightarrow \quad \sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{10000} = 10000 \cdot \sqrt{10000} \cdot \tfrac{2}{3} + \tfrac{1}{2} \cdot \sqrt{10000} =$

$= 666716\tfrac{2}{3}$